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ON THE ANALOGUES AND GENERALIZATIONS OF SQUIRE'S THEOREM*

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Assertions consistent with Squire's theorem /1/ based on the simultaneous or separate consideration of the factors responsible for the non-linearity of the motion and the non-stationarity of the fluid flow whose stability is investigated, are studied. The results may be of interest from the point of view of the practical applications, and can also be used to solve the problem of the degree of generality of the reasons governing the existence of assertions of the type of Squire's theorem.

In the best known, classical version of the linear problem of the stability of the plane parallel flow of an incompressible fluid with constant density and viscosity parameters, Squire's theorem consists, in fact, of two assertions.

A. Equations determining the dynamics of three-dimensional perturbations (in the form of normal waves) can be reduced, by means of some transformation, to the equations for plane perturbations. This means that the three-dimensional perturbation and the corresponding plane perturbation are stable or unstable simultaneously.

B. The expression for the coefficient of kinematic viscosity occurring in the course of transformation A, is used as the basis for drawing conclusions on the loss of stability under plane perturbations when the viscosity is greater (a smaller Reynolds number of the basic flow) than in the case of three-dimensional perturbations.

The analogues of Squire's theorem for a compressible /2/ or stratified /3/ fluid are restricted to assertion A. In the gas-dynamic formulation the situation with assertion B can become reversed, with the three-dimensional perturbations becoming more "dangerous" than the plane ones. The absence, in general, of any indication of which are the most dangerous perturbations, makes these analogues less useful, although they are also undoubtedly important from the point of view of simplifying the problem and reducing it to the "canonical" formulation concerning the stability of a plane parallel flow under plane perturbations.

1. Translationally invariant motions. We consider the motion of a viscous, incompressible fluid of constant density, between two parallel planes $\zeta=\pm H$, in a Cartesian coordinate system ξ, η, ζ. The equations of motion of the fluid for the velocity components u_0, v_0, w_0, pressure p and the boundary conditions, can be written in the form

$$D_0u_0 = -p_\xi + \nu\Delta u_0 + f_0(\zeta, t) \tag{1.1}$$

$$D_0v_0 = -p_\eta + \nu\Delta v_0 + g_0(\zeta, t),\ D_0w_0 = -p_\zeta + \nu\Delta w_0$$

$$u_{0\xi} + v_{0\eta} + w_{0\zeta} = 0;\quad D_0 = \frac{\partial}{\partial t} + u_0\frac{\partial}{\partial\xi} + v_0\frac{\partial}{\partial\eta} + w_0\frac{\partial}{\partial\zeta}$$

$$\zeta = \pm H,\ u_0 = U^\pm(t),\ v_0 = V^\pm(t),\ w_0 = 0 \tag{1.2}$$

Here ν is the constant coefficient of kinematic viscosity, the density $\rho\equiv 1$, Δ is the three-dimensional Laplace operator. The indices with independent variables denote the corresponding partial derivatives, the field $\mathbf{f}(\zeta, t)$ of external mass forces depends on the coordinate ζ and time t only. We will assume, without loss of generality, that the force component ζ is equal to zero: $\mathbf{f} = (f_0, g_0, 0)$.

After specifying the suitable initial conditions for (1.1) and (1.2), we obtain the initial boundary value problem, and we shall study the properties of its solution.

Eqs.(1.1) and boundary conditions (1.2) are invariant under displacements along the ξ and η axes, therefore it makes sense to consider translationally invariant solutions possessing the same symmetry. In order to describe the motions belonging to this class, we introduce the following unit vector defining the direction:

$$\mathbf{n} = \mathbf{n}(\theta) = (-\sin\theta,\ \cos\theta,\ 0),\ 0\leqslant\theta\leqslant\pi \tag{1.3}$$

lying in the $\xi\eta$ plane and making an angle θ with the η axis. We denote by T_θ the set of all translationally invariant solutions in which the hydrodynamic fields ($\mathbf{u}$ and ∇p) remain unchanged at any instant t when the coordinate system is displaced along the vector $\mathbf{n}(\theta)$. It

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is clear that T_θ contains a set of plane motions for which $\mathbf{u}\cdot\mathbf{n}=0$.

In the case of motions belonging to the class T_θ we eliminate the dependence of the hydrodynamic fields on one of the spatial variables, by rotating the coordinate system about the ζ axis by an angle θ. In the new axes of the Cartesian coordinate system x, y, z we denote the velocity components by u, v, w, and we have the following relations:

$$x=\xi\cos\theta+\eta\sin\theta,\ y=-\xi\sin\theta+\eta\cos\theta,\ z=\zeta \qquad (1.4)$$
$$u=u_0\cos\theta+v_0\sin\theta,\ v=-u_0\sin\theta+v_0\cos\theta,\ w=w_0$$

The following functional relations also hold:

$$u=u(x,z,t),\ v=v(x,z,t),\ w=w(x,z,t),\ p=p(x,z,t)-A(t)\,y \qquad (1.5)$$

In the expression for the pressure given in (1.5), the term containing the function $A(t)$ corresponds to the already mentioned invariance of the field ∇p (and not of p). In x, y, z coordinates problem (1.1), (1.2) for the motions (1.5) reduces to a form which can be conveniently written as a sequence of two problems.

Problem 1. (the "plane" problem)

$$Du=-p_x+\nu\Delta_2 u+f,\ \ Dw=-p_z+\nu\Delta_2 w \qquad (1.6)$$
$$u_x+w_z=0;\ \ D=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z},\ \ \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$$
$$z=\pm H,\ u=U^\pm,\ w=0$$

Problem 2. (the "one-dimensional" problem)

$$Dv=A+\nu\Delta_2 v+g;\ z=\pm H,\ v=V^\pm \qquad (1.7)$$

Here f and g are the x- and y-components of the force $\mathbf{f}$, expressed in terms of f_0 and g_0 (1.1) as in (1.4).

Problem 1 has two special features: 1) the component v does not appear in (1.6), therefore the latter can be solved independently of problem 2; 2) problem 1 is mathematically identical with the problem of describing the plane motions. In problem 2 the components u and w participate through the operator D. Therefore, the unknown function $v(x,z,t)$ appearing in (1.7) can be determined only after problem 1 has been solved. Below we shall show that the facts mentioned above form the basis of Squire's theorem, as well as of some of its analogues and generalizations.

2. Perturbations in slippage flows.

The flows with layers slipping relative to each other $\zeta=\text{const}$ (called here slippage flows)

$$u_0=U_0(\zeta,t),\ v_0=V_0(\zeta,t),\ w\equiv 0 \qquad (2.1)$$
$$p=P_0(\xi,\eta)\equiv A_0(t)\,\xi-B_0(t)\,\eta$$

correspond to the exact solutions of problem (1.1), (1.2). The flows have velocity components $U_0(\zeta,t), V_0(\zeta,t)$ and functions of time $A_0(t), B_0(t)$ determining the pressure gradient, which satisfy the relations

$$U_{0t}=A_0+\nu U_{0\zeta\zeta}+f_0,\ V_{0t}=B_0+\nu V_{0\zeta\zeta}+g_0 \qquad (2.2)$$

The special case of (2.1), (2.2) with

$$V_0\equiv g_0\equiv B_0\equiv 0 \qquad (2.3)$$

is called plane-parallel flow.

The investigation of the stability of the flow (2.1), (2.2) reduces to a study of the behaviour of the perturbations (denoted here by primes)

$$u_0=U_0+u_0',\ v_0=V_0+v_0',\ w_0=w_0',\ p=P_0+p' \qquad (2.4)$$

and the boundary conditions are $\zeta=\pm H,\ u_0'=v_0'=w_0'=0$.

Let the perturbation fields be translationally invariant with respect to some direction $\mathbf{n}(\theta)$ (1.3). It is clear that the complete solutions of (2.4) also belong to the class T_θ. From the arguments presented in Sect.1 we have

Assertion 1. The problem of describing the translationally invariant perturbations of the slippage flow (2.1), (2.2) can be separated into two problems, which can be solved consecutively.

Problem 1a.

$$Du'+U_zw'=-p_x'+\nu\Delta_2 u',\ Dw'=-p_z'+\nu\Delta_2 w' \qquad (2.5)$$

$$u_x' + w_z' = 0; \quad D = \frac{\partial}{\partial t} + (U + u')\frac{\partial}{\partial x} + w'\frac{\partial}{\partial z}$$
$$z = \pm H, \ u' = w' = 0$$

Problem 2a.

$$Dv' + V_z w' = \nu \Delta_2 v'; \ z = \pm H, \ v' = 0 \tag{2.6}$$

The absence of a zero subscript on the unknown functions in (2.5), (2.6) means that the x, y, z (1.4) system of coordinates is used.

The proof of the Assertion 1 follows from relations (1.6), (1.7).

Introducing into the plane parallel flow (2.1)-(2.3) the notation

$$u' = u_1 \cos\theta, \ w' = w_1 \cos\theta, \ v' = v_1 \sin\theta$$
$$p' = p_1 \cos^2\theta, \ \nu = \nu_1 \cos\theta, \ \tau = t\cos\theta$$

we obtain, from Assertion 1,

Assertion 2. The problem of describing translationally invariant perturbations (belonging to the class T_θ when $\theta \neq \pi/2$) in plane parallel flow with a velocity profile $U_0(\zeta, t)$, can be represented in the form of two successively solvable problems.

Problem 1b.

$$D_1 u_1 + U_{0z} w_1 = -p_{1x} + \nu_1 \Delta_2 u_1, \ D_1 w_1 = -p_{1z} + \nu_1 \Delta_2 w_1 \tag{2.7}$$
$$u_{1x} + w_{1x} = 0; \quad D_1 = \frac{\partial}{\partial \tau} + (U_0 + u_1)\frac{\partial}{\partial x} + w_1 \frac{\partial}{\partial z}$$
$$z = \pm H, \quad u_1 = w_1 = 0$$

Problem 2b.

$$D_1 v_1 - U_{0z} w_1 = \nu_1 \Delta_2 v_1; \ z = \pm H, \ v_1 = 0 \tag{2.8}$$

The exception $\theta = \pi/2$ corresponds to the invariance of the perturbation fields along the direction of the velocity of the basic flow. Separate investigation leads to the formulation (2.5), (2.6) with $U \equiv 0, V = U_0$, and relations (2.5) correspond here to a simple problem of the decay of plane perturbations of the state of rest.

3. The unimportance of "one-dimensional" problems. The fundamental step that follows consists of demonstrating the unimportance of the separated "one-dimensional" problems (2.6) or (2.8). To express it more accurately, we have to show that the one-dimensional problems contain no additional instabilities compared with the "plane" problems (2.5) or (2.7). Having proved this fact, we find that the analogues and generalizations of Squire's theorem follow as straightforward and obvious corollaries of Assertions 1 and 2.

Assertion 3. We consider, in the strip

$$\Pi = \{(x, z, \tau): -\infty < x < \infty, \ -H < z < H, \ \tau > 0\}$$

the linear parabolic equation

$$\mu L\varphi \equiv \mu \Delta_2 \varphi + a(x, z, \tau)\varphi_x + b(x, z, \tau)\varphi_z - \varphi_\tau = f(x, z, \tau) \tag{3.1}$$

where μ is a positive constant; a, b and f are functions defined on Π, continuous in all variables and subject to the following boundary constraints:

$$\sup_\Pi |b| < C_1, \ \sup_\Pi |f| < C_2 \tag{3.2}$$

Let $\varphi(x, z, \tau)$ be the classical solution of (3.1) in Π, satisfying the boundary condition

$$\varphi(x, \pm H, \tau) = 0 \tag{3.3}$$

and taking the following initial values:

$$\varphi(x, z, 0) = \varphi_0(x, z); \qquad \sup_{\substack{-\infty<x<\infty \\ -H<z<H}} |\varphi_0(x, z)| < C_3 \tag{3.4}$$

Moreover, let the solution $\varphi(x, z, \tau)$ satisfy one of the following conditions:

1°, When $|x| \to \infty$, the function $\varphi(x, z, \tau) \to 0$ uniformly in z and $\tau \in [0, \tau^*]$, where τ^* is an arbitrary positive number.

2°. The function $\varphi(x, z, \tau)$ is periodic in x, i.e. there exists a value $0 < X < \infty$, such that $\varphi(x, z, \tau) = \varphi(x + X, z, \tau)$ for any x, z, τ of Π.

Then the following estimate holds:

$$|\varphi(x,z,\tau)| \leqslant \frac{\varkappa_1+\varkappa_2}{\mu} C_2 + \varkappa_1 C_3 \exp\left\{-\frac{\mu\tau}{\varkappa_2}\right\} \tag{3.5}$$

$$\varkappa_1 \equiv (1-e^{-\lambda H})^{-1},\ \varkappa_2 \equiv e^{3\lambda H}$$

$$2\lambda \equiv C_1\mu^{-1} + \sqrt{C_1^2\mu^{-2}+4} + 1$$

The proof utilizes the technique given in /4/. We consider the function $R(z) = e^{2\lambda H} - e^{\lambda z}$, where λ is an arbitrary positive constant. If we choose $2\lambda > \mu^{-1}C_1 + \sqrt{C_1^2\mu^{-2}+4}$, then by virtue of (3.2)

$$LR = -(\lambda^2 + b\mu^{-1}\lambda)e^{\lambda z} < -e^{\lambda z} \leqslant -e^{-\lambda H} \equiv \delta \tag{3.6}$$

everywhere in Π.

We introduce the auxiliary quantities

$$\delta_0 \equiv \min_{-H\leqslant z\leqslant H} R = e^{2\lambda H} - e^{\lambda H},\quad \delta_1 \equiv \max_{-H\leqslant z\leqslant H} R = e^{2\lambda H} - e^{-\lambda H}$$

and a function

$$\psi(x,z,\tau) = \left(\frac{\varepsilon}{\delta_0} + \frac{\varepsilon}{\delta} + \frac{A_1}{\delta_0}e^{-\gamma\tau}\right)R(z)$$

defined in Π, where ε, A_1, γ are any positive constants and $\tau \geqslant 0$. Having chosen $\gamma = \mu\delta\delta_1^{-1}$, we can easily confirm that

$$\psi(x,z,0) > A_1,\ \psi(x,\pm H,\tau) > \varepsilon,\ L\psi < -\varepsilon \tag{3.7}$$

The choice $A_1 = C_3$ yields, by virtue of (3.4), $\psi(x,z,0) > \varphi(x,z,0)$. We also have $\psi > \varphi$ at the boundaries $z = \pm H$ by virtue of (3.3), (3.7).

Now let M be a set of values $\sigma \geqslant 0$, such, that the inequality

$$\varphi < \psi \tag{3.8}$$

holds in the strip

$$\Pi_\sigma = \{(x,z,\tau): -\infty < x < \infty,\ -H \leqslant z \leqslant H,\ 0 \leqslant \tau \leqslant \sigma\}$$

This, together with condition 1° (or 2°) guarantees that $t^* = \sup\{M\} > 0$.

Next we shall show that the inequality (3.8) holds for all values of $\tau > 0$. Let us assume the opposite, that t^* is a finite quantity. Then the function $S \equiv \psi - \varphi$ will be strictly positive when $\tau < t^*$, and negative when $\tau = t^*$, i.e. there exists a point (x_0, z_0, t^*) such that $S(x_0, z_0, t^*) = 0$. From (3.3), (3.7) and condition 1° (or 2°) it follows that $-H < z_0 < H$, $-\infty < x_0 < \infty$. If the function S is regarded as a function of two variables x and z, then $\tau = t^*(x_0, z_0)$ will be its minimum. Therefore the condition $LS \geqslant 0$ holds at the point (x_0, z_0, t^*). At the same time, the following inequality holds for all $\tau > 0$:

$$LS = L\psi - L\varphi < -\varepsilon - f\mu^{-1}$$

from which it follows, after choosing $\varepsilon = C_2\mu^{-1}$, that $LS < 0$. The resulting contradiction means that t^* becomes infinite and inequality (3.8) holds for all $\tau \geqslant 0$.

We prove the inequality $-\psi < \varphi$ in exactly the same manner, therefore the estimate

$$|\varphi(x,z,\tau)| < \psi(x,z,\tau) \leqslant \frac{C_2}{\mu}\left(\frac{\delta_1}{\delta} + \frac{\delta_1}{\delta_0}\right) + C_3\frac{\delta_1}{\delta_0}\exp\left\{-\frac{\mu\delta\tau}{\delta_1}\right\}$$

holds and (3.5) follows from it.

The one-dimensional problem (2.6), (2.8) can be reduced to the form (3.1)-(3.4) by a straightforward change of notation. Thus for problem 2b we write

$$\mu \equiv \nu_1,\ a \equiv -U_0 - u_1,\ b \equiv -w_1,\ f \equiv -U_{0z}w_1$$

and then the inequality (3.5) yields an estimate for the perturbation in terms of its initial values (constant C_3) and in terms of the perturbation of the z component of the velocity (constants C_1 and C_2). A direct consequence of this is

Assertion 4. Let us assume that the plane parallel flow (2.1)-(2.3) is Lyapunov stable for the solution of problem 1b, so that for any number $\varepsilon > 0$ there exists $\delta > 0$, such that when the inequality $|w_1| < \delta$ holds at $\tau = 0$, then we have the inequality $|w_1| < \varepsilon$, which holds at any instant of time $\tau > 0$. Then for any value $\varepsilon_0 > 0$ there exists $\delta_0 > 0$ such that if the inequalities $|v_1|, |w_1| < \delta_0$ hold at $\tau = 0$, then we have the estimates $|v_1|, |w_1| < \varepsilon_0$ which hold at any instant of time $\tau > 0$.

In other words, the Lyapunov stability of the zero solution of the plane problem (2.7) implies the stability for the complete problem (2.7), (2.8). Moreover, the stability of solutions of (2.8) does not require the stability of the component u_1 in (2.7).

4. Fundamental results. We can now formulate the following results stemming directly

from Assertions 1-4.

Assertion 5. The slippage flow (2.1), (2.2) with velocity components $U_0(z, t)$, $V_0(z, t)$ and viscosity ν is stable under perturbations belonging to the class T_θ if and only if the plane parallel flow (2.1)-(2.3) with the profile

$$U(z, t) \equiv U_0(z, t) \cos\theta + V_0(z, t) \sin\theta$$

and the same value of the viscosity is stable under plane perturbations.

Assertion 6. The plane parallel flow (2.1)-(2.3) with velocity profile $U_0(z, t)$ and viscosity ν is stable under perturbations belonging to class T_θ if and only if another plane parallel flow with the profile

$$u = U_1(z, t) \equiv U_0(z, t/\cos\theta) \tag{4.1}$$

and higher value of viscosity $\nu_1 \equiv \nu/\cos\theta$ is stable under plane perturbations.

Both assertions refer to type A, and can be regarded as analogues of Squire's theorem within the framework of the class of translationally invariant motions. In the special case of stationary flows, Assertion 6 yields.

Assertion 7. If a stationary, plane parallel flow is stable under plane perturbations, then it will be even more stable under any translationally invariant perturbations. This guarantees the stability on the set T of the perturbations obtained by unifying the classes T_θ for all values of θ.

The above result refers to the same type as Squire's theorem (set of Assertions A and B). The difference lies in the conditional character of Assertion 7. Non-linear stability is not guaranteed for all perturbations, but only for those belonging to class T. At the same time, Squire's theorem itself can be obtained from Assertion 7 after linearization. The point is that the principal simplification of the linear problem (the presence of the principle of superposition) makes it possible to construct any other solutions from the translationally invariant solutions. We may add here, for clarity, that normal waves (perturbations of the flow (2.1)-(2.3) proportional to $\exp[i(k\xi + l\eta - \lambda t)]$), usually studied in the classical formulation of Squire's theorem, represent a special case of the translationally invariant solutions.

Thus we see, at the basis of Squire's theorem and of other assertions of this type, there lies the "separation" of the one-dimensional problems, characteristic of the translationally invariant motions, which occurs in a number of formulations falling considerably outside the framework of traditional formulations.

Notes. 1°. In the linear limit Squire's theorem is obtained from Assertion 7, together with the complementary statement /5/ on the unimportance of the part of the spectrum omitted by Squire. Namely, if we take the perturbation in the form of normal waves, then, using Assertion 3 we obtain at once the estimate $\mathrm{Im}\,\lambda < 0$ for the spectrum of the one-dimensional problem (resulting from (2.8)).

2°. The linearization of the equations of motion does not enable Assertion 6 to be amplified for non-stationary flows, so that it will distinguish the most dangerous perturbations (see Assertion B). This is due to the fact that in case of the perturbations belonging to class T_θ the fictitious increase in the viscosity is accompanied by a simultaneous "increase" in the degree of instability of the flow (4.1). Thus, if the profile $U_0(z, t)$ is harmonic in t with frequency ω, then the frequency with which the fictitious profile $U_1(z, t)$ will change is equal to $\omega/\cos\theta$. The answer to the problem of the most "dangerous" perturbations depends here on the distribution of the neutral curve in the plane of the Reynolds and Strouhal numbers.

3°. In the case of an ideal fluid the splitting of the equations of motion (the results of Sect.1 and 2 at $\nu = 0$) remains true, but the unimportance of the one-dimensional problems can no longer be proved. For example, in the case of stationary slippage flow Eq.(2.6) is written in the form $D(V + v') = 0$. From the point of view of the plane problem (2.5) this equation means that the quantity $V + v'$ is retained in every fluid particle. Since the perturbations of the z component of velocity do not, in general, decay when $\nu = 0$, the finiteness of the displacement of these particles may lead to instability of the "kinematic" type. Such instability was shown in the linear approximation earlier in /6/, Sect.2.

4°. An interesting situation arises in the case of the stationary, plane parallel flows of an ideal fluid. In the class T_θ for any value of θ we have, for the convex profile ($U_{0zz} \neq 0$), a non-linear stability in the sense of /7, 8/ for the plane problem (2.7), while at the same time we may have no stability for the one-dimensional problem. In other words, we have stability in the norm /7, 8/ for the components u, w, but not for v.

5°. In the case of linear wave perturbations and a stationary profile of the basic flow, the analogues of Assertion 5 were given in /9, 10/.

6°. Results similar to those given above can be obtained for the flows of an incompressible fluid of the type (2.1), with the density and/or viscosity depending on the z coordinate, and also for a compressible fluid.

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DEVELOPMENT OF THE TOLLMIN - SCHLICHTING WAVE IN A BOUNDARY LAYER, ON A PLATE*

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The development of three-dimensional perturbations of constant frequency in a boundary layer on a semi-infinite plate is studied within the framework of the Navier-Stokes (NS) equations for an incompressible fluid. A case in which the Tollmin-Schlichting (TS) /1, 2/ wave has reached a point on the plate corresponding to the lower branch of the neutral stability curve (NSC), obtained by solving the eigenvalue problem for the Orr-Sommerfeld equation, is discussed. An asymptotic solution of the non-linear NS equations at large Reynolds numbers in given. According to the result obtained, first we have a non-linear process taking place within the NSC near its lower branch, for the separated TS wave with an amptitude that is not too small, leading to gradual reduction in the wave amplitude. Since the Blasius boundary layer is not parallel, the process changes when the amplitude increases. Thus the point at which the amplitude of the TS wave is at a minimum, lies within the loop of the NSC. Therefore, when the experiment is compared with the linear theory based on the Orr-Sommerfeld equation, the theory must be corrected.

Non-linear effects in the theory of the TS waves were first studied in /3/, where an equation for the wave amplitude was given. A strict proof of the amplitude equation was obtained later in /4/ for the case of perturbations periodic in the longitudinal direction of the coordinate. The effect of non-parallelism of the flow on the coefficients of this equation was studied in /5/. The amplitude equation was analysed, without taking into account

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